Lecture 1

Intro:
- email: [redacted] eburkard+Teaching@nd.edu
- webpage: nd.edu/~eburkard
- course master page: nd.edu/~taylor/Math20550
- Grading
- Midterm grade based ENTIRELY on Exam I.
- Homework
  - How to sign up
  - Make sure to write out solutions to homework to make sure you have a record of how you solved the problems.
- Office hours: MWF 16:30–17:30, Hurley 295
  or by appointment.
- Questions?

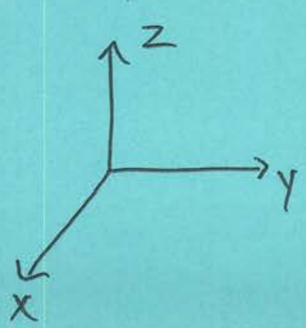
Section 12.1 - 3D Coordinates

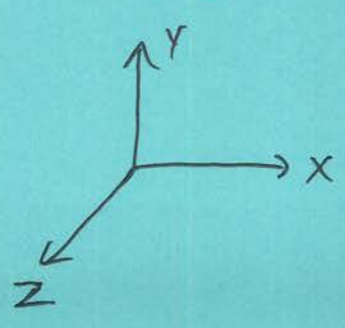
In Calc I & II, you have been studying functions of one variable. In Calc III, we will move beyond that, and to do so we will need more variables.

$\mathbb{R}^3$: We will usually work in 3-D space in this class, which requires 3 variables. We use coordinates (x, y, z) and describe a point by (a, b, c), where a is the x-coordinate, b the y-coordinate, and c the z-coordinate. Just as in the xy-plane, where you move counterclockwise from the positive x-axis to the positive y-axis (usually, at least), the orientation of the x-, y-, and z- axes matter. It follows the Right Hand Rule:

- Index finger points along x-axis
- Middle finger points along y-axis
- Thumb points along z-axis

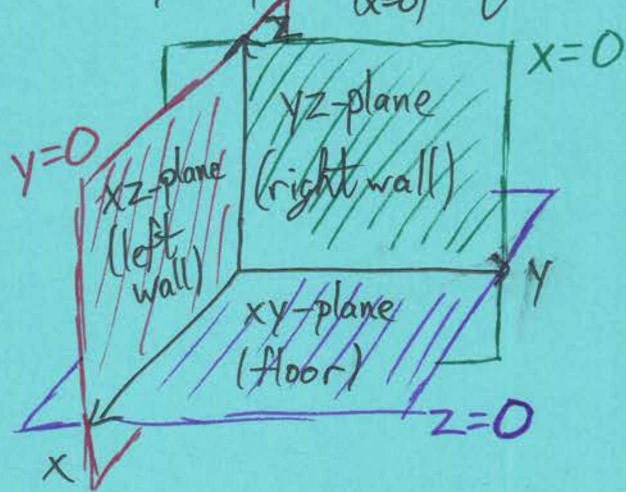
Examples:





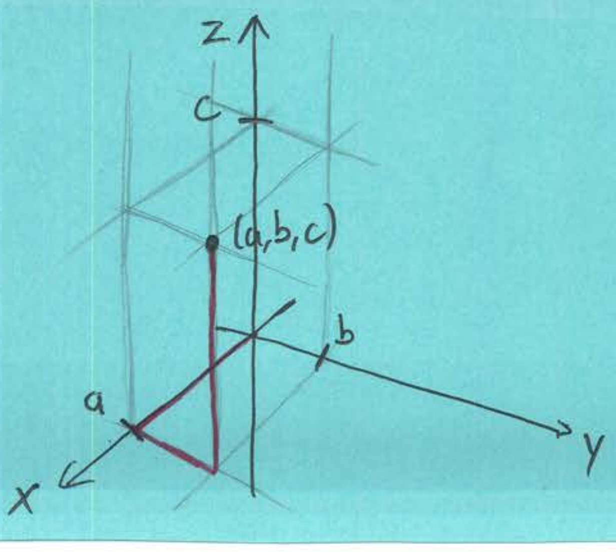
In $\mathbb{R}^3$, there are 3 <u>coordinate planes</u>:

- xy-plane (floor) ($z=0$)
- xz-plane ($y=0$) (left wall)
- yz-plane ($x=0$) (right wall)



These coordinate planes divide $\mathbb{R}^3$ into <u>octants</u>.
The octant where $x, y, z > 0$ is called the <u>first octant</u>.
(That's the only one that's named, there isn't a "natural" way to order the rest.)

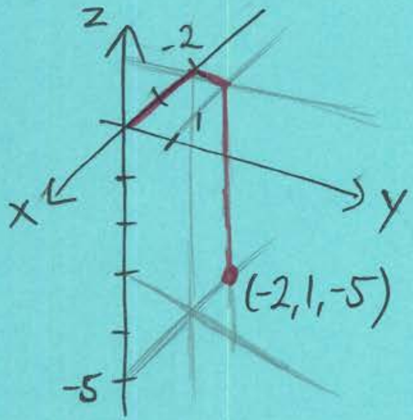
<u>Points</u>: A point P in $\mathbb{R}^3$ is described by an ordered triple: (a,b,c). We graph it here:



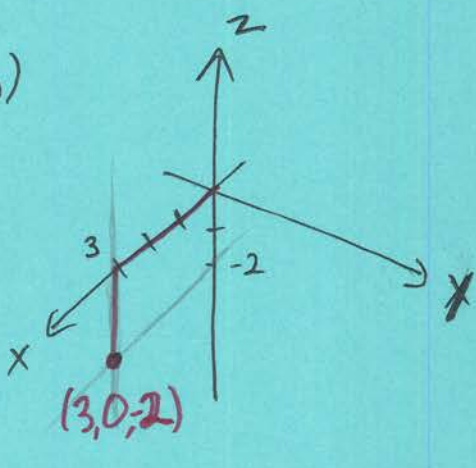
Actual examples:

a) $(-2, 1, -5)$, b) $(3, 0, -2)$ c) $(1, -2, 3)$

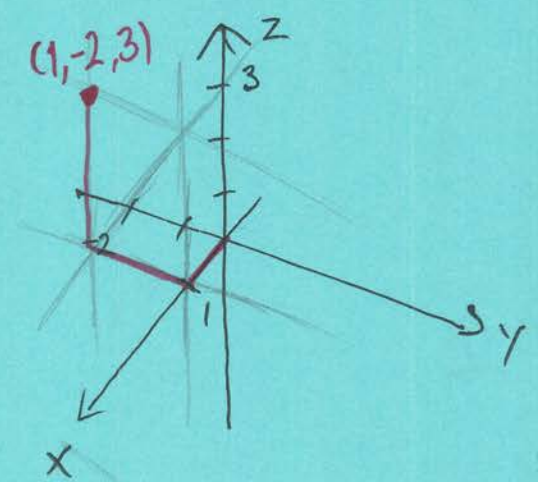
a)



b)

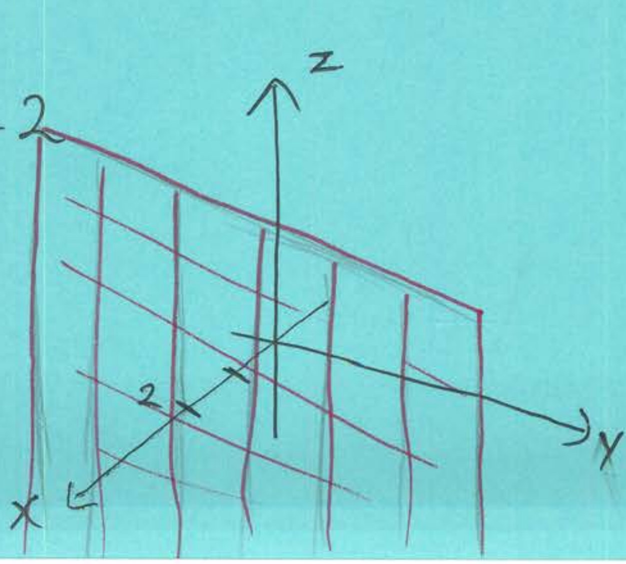


c)

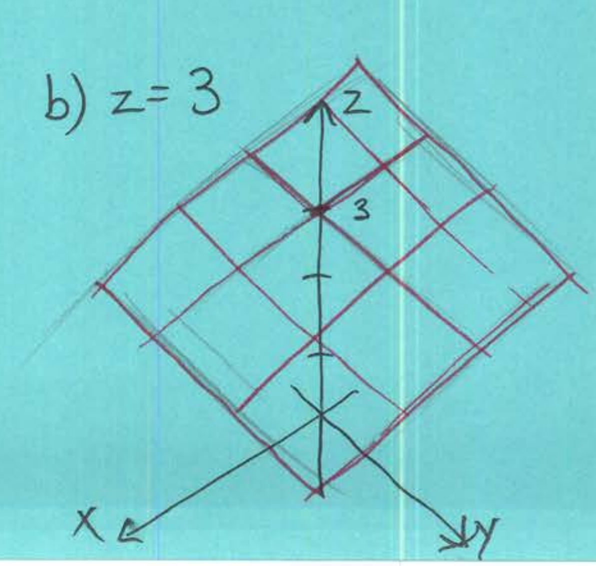


In $\mathbb{R}^2$, when we graphed $x = 2$, $y = -1$, or other things like that, we got lines; in $\mathbb{R}^3$, these equations give planes:

Ex: a) $x = 2$



b) $z = 3$

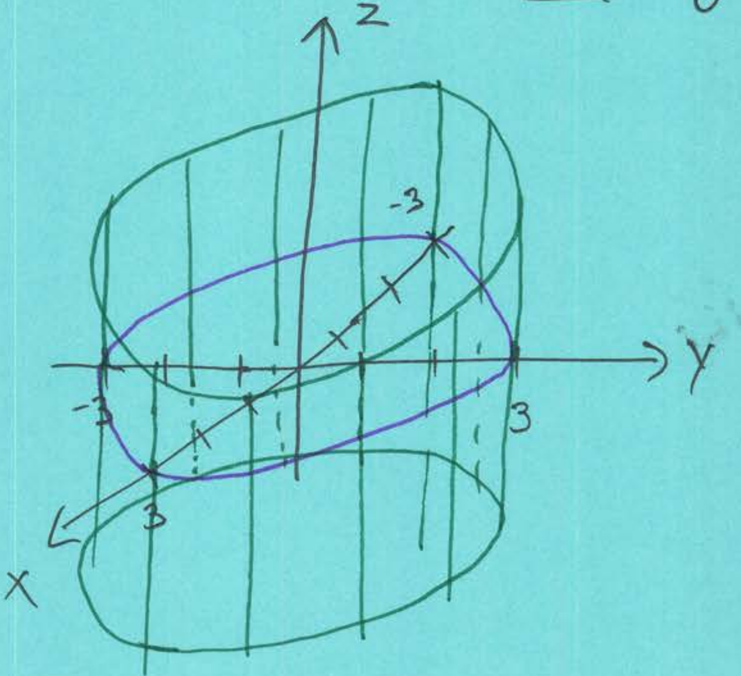


Other surfaces

What does the equation $x^2+y^2=9$ represent in $\mathbb{R}^3$?

Ans: In $\mathbb{R}^2$, this is a circle of radius 3, but in $\mathbb{R}^3$ we have the z-variable, which this equation does not restrict, hence z can be anything!

Graph:

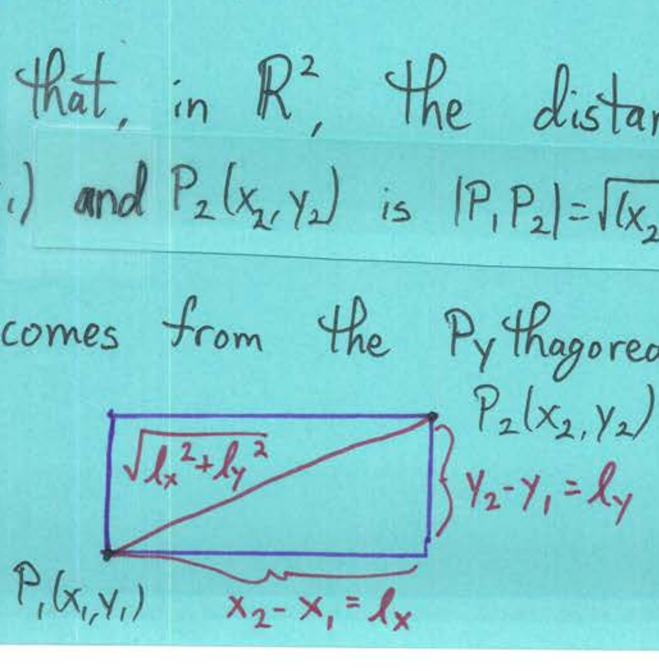


a cylinder along the z-axis with radius 3.

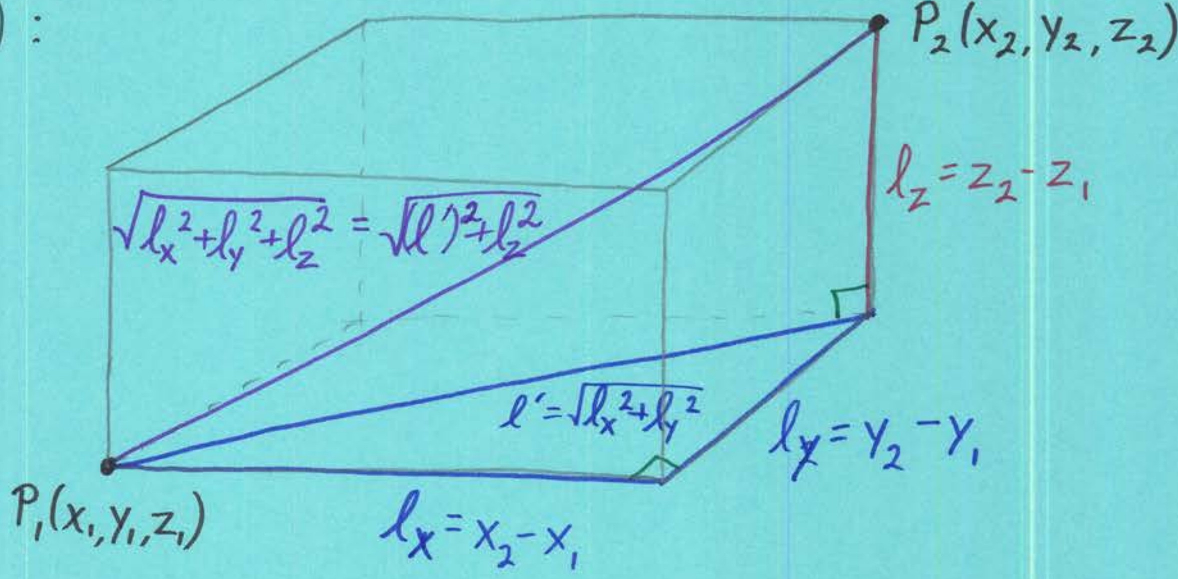
Distance in $\mathbb{R}^3$:

Recall that, in $\mathbb{R}^2$, the distance between points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is $|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

This comes from the Pythagorean theorem:



The distance formula in $\mathbb{R}^3$ is obtained in a similar fashion. Take two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$:



So, $|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

<u>Ex</u>: Describe the set of points which are a distance [redacted] r away from the point $C(h, k, l)$.

<u>Sol</u>: Let an arbitrary such point be called $P(x, y, z)$. The the distance formula gives

$$|PC| = \sqrt{(x-h)^2 + (y-k)^2 + (z-l)^2} = [redacted]\ r$$

Square both sides and get

$$\boxed{(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2}$$

[redacted] Equation of a <u>sphere</u> with <u>center</u> (h, k, l) and <u>radius</u> r.

Ex: Determine whether

$$x^2+y^2+z^2+8x-6y+2z+17=0$$

is a sphere. If so, give the center and radius.

Sol: Complete the square:

$$x^2+8x+16+y^2-6y+9+z^2+2z+1=-17+16+9+1$$

$$\Longrightarrow (x+4)^2+(y-3)^2+(z+1)^2=9$$

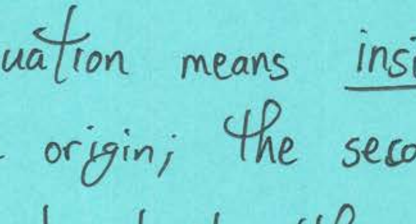
This is a sphere of radius 3 with center $(-4,3,-1)$.

Ex: What region do the inequalities

$$x^2+y^2+z^2\leq 4 \quad \& \quad x^2+y^2\geq 1 \quad \& \quad z\geq 0$$

describe?

Sol: The first equation means <u>inside</u> the sphere of radius 2, centered at the origin; the second means <u>outside</u> the cylinder of radius 1, about the z-axis; and the third means <u>above</u> the xy-plane. So, graphically, we have the solid:



(A half-dome with a hole drilled through the top.)